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Probabilistic Progressive Buckling of Trusses

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A three-bay, space, cantilever truss is probabilistically evaluated to describe progressive buckling and truss collapse in view of the numerous uncertainties associated with the structural, material, and load variables that describe the truss. Initially, the truss is deterministically analyzed for member forces, and members in which the axial force exceeds the Euler buckling load are identified. These members are then discretized with several intermediate nodes, and a probabilistic buckling analysis is performed on the truss to obtain its probabilistic buckling loads and the respective mode shapes. Furthermore, sensitivities associated with the uncertainties in the primitive variables are investigated, margin of safety values for the truss are determined, and truss end node displacements are noted. These steps are repeated by sequentially removing buckled members until onset of truss collapse is reached. Results show that this procedure yields an optimum truss configuration for a given loading and for a specified reliability.

Nomenclature

E	= modulus of elasticity, Mpsi
H	= longitudinal load, lb
$[K]$	= stiffness matrix
$[K_g]$	= geometric stiffness matrix
M	= twisting moment, lb in.
r_i	= inner radius, in.
r_o	= outer radius, in.
V	= vertical load, lb
X, Y, Z	= Cartesian coordinate axes
λ	= eigenvalue
ϕ	= eigenvector

Introduction

IT is customary to evaluate the structural integrity of trusses by using deterministic analysis techniques and appropriate load/safety factors. Traditionally, these factors are an outcome of many years of analytical, as well as experimental, experience in the areas of structural mechanics and design. Load factors are used to take into account uncertainties in many different operating conditions, including the maximum loads, and safety factors are also used to account for unknown effects in analysis assumptions, fabrication tolerances, and material properties.

An alternative to the deterministic approach is the probabilistic structural analysis method (PSAM).[1] PSAM formally accounts for various uncertainties in primitive variables (fundamental parameters describing the structural problem) and uses different distributions, such as the Weibull, normal, log normal, etc., to define these uncertainties. Furthermore, PSAM assesses the effects of these uncertainties on the scatter of structural responses (displacements, frequencies, eigenvalues). Thus, PSAM provides a more realistic and systematic way to evaluate structural performance and durability. A part of PSAM is a computer code called numerical evaluation of stochastic structures under stress (NESSUS) that provides a choice of solutions for static, dynamic, buckling, and nonlinear analysis.[2,3]

In the recent past, NESSUS has been used for the analysis of Space Shuttle Main Engine (SSME) components. Representative examples include a probabilistic assessment of a mistuned bladed disk assembly[4] and an evaluation of the reliability and risk of a turbine blade under complex service environments.[5] Furthermore, NESSUS has also been used to computationally simulate and probabilistically evaluate a cantilever truss typical for space-type structures[6] and to quantify the uncertainties in the structural responses (displacements, member axial forces, and vibration frequencies). The objective of this paper is to develop a methodology and to perform probabilistic progressive buckling assessment of space-type trusses using the NESSUS computer code.

Fundamental Approach and Considerations

One of the major problems encountered in the analysis of space-type trusses is to develop a stable and optimum configuration for given loading conditions and to be able to probabilistically analyze them to take into account the uncertainties in the primitive variables typical for space environment conditions. Currently, it is a practice to design these trusses with cross bracings, thereby increasing the overall weight of the truss, the cost of fabrication, and the effort to deploy in space. Furthermore, the currently available methods and programs do not easily allow us to identify any local instability in any of the internal members of the truss during probabilistic buckling (eigenvalue) analysis and to calculate overall margins of safety of the truss. Therefore, using the NESSUS code, we have developed a methodology for the probabilistic progressive buckling as described hereafter.

Finite Element Model

A three-dimensional, three-bay cantilever truss is computationally simulated using a linear isoparametric beam element based on the Timoshenko beam equations. The element is idealized as a two-noded line segment in three-dimensional space. The cantilever truss is assumed to be made from hollow circular pipe members. The members are made up of wrought aluminum alloy (616-w) with modulus of elasticity E equal to 10 Mpsi. The outer and inner radii r_o and r_i of the tube are 0.5 and 0.4375 in., respectively. All six degrees of freedom are restrained at the fixed end (left side) nodes. Each bay of the truss is 5 ft

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wide, 8 ft long, and 6 ft high (Fig. 1). The overall length of the truss is 24 ft. Six vertical and two longitudinal loads are applied. In addition, twisting moments are applied at the truss-end nodes. The directions of the forces and moments are shown in Fig. 1, and mean values are given in Table 1. The applied loads and moments are selected to represent anticipated loading conditions for a typical space truss.

Buckling of Columns

In slender columns, a relatively small increase in the axial compressive forces will result only in axial shortening of the member. However, the member suddenly bows out sideways if the load level reaches a certain critical level. Large deformations caused by increased induced bending moment levels may lead to the collapse of the member. On the other hand, tension members as well as short stocky columns fail when the stress in the member reaches a certain limiting strength of the material.

> According to Chajes,[7] buckling, however, does not occur as a result of the applied stress reaching a certain predictable strength of the material. Instead, the stress at which buckling occurs depends on a variety of factors, including the dimensions of the member, the way in which the member is supported, and the properties of the material out of which the member is made.

Chajes also describes the concept of neutral equilibrium that is being used to determine the critical load of a member such that at this load level the member can be in equilibrium both in the straight and in a slightly bent configuration. Furthermore, the Euler load (buckling load or critical load) is the smallest load at which a state of neutral equilibrium is possible or the member ceases to be in stable configuration. This previous definition of buckling load is used to identify the probable truss

Table 1 Primitive variables and uncertainties for probabilistic structural analysis of a space truss

Primitive variables	Distribution type	Mean value	Scatter, ±%
Geometry	Normal	60 in.	0.5
Width	Normal	96 in.	0.1
Length		192 in.	0.1
		288 in.	0.1
	Normal	72 in.	0.2
Height			
Loads	Lognormal	20 lb	6.3
Vertical	Lognormal	20 lb	2.5
Longitudinal	Lognormal	50 lb in.	6.3
Twisting moment			
Material property	Normal	10 Mpsi	7.5
Modulus			
Tube radii			
Outer radius	Normal	0.5 in.	7.5
Inner radius	Normal	0.44 in.	7.5

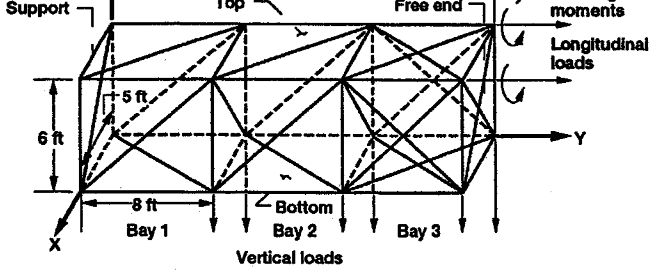

Fig. 1 Solar array panels mast of typical truss.

members that contribute to the progressive buckling behavior of the cantilever truss.

Probabilistic Model

The primitive variables listed in the Nomenclature are considered in the probabilistic analysis. It is possible that those primitive variables will vary continuously and simultaneously due to extreme changes in the environment when such trusses are used in upper Earth orbit for space station type structures. The normal distribution is used to represent the uncertainties in E, r_o, r_i, and X, Y, Z coordinates. The applied loads and moments are selected to represent an anticipated loading for a typical space truss. The scatter in these are represented by log-normal distributions. Initially, the NESSUS/finite element methods (FEM) module is used to deterministically analyze the truss at the mean values of each primitive variables. In the subsequent probabilistic analysis, each primitive variable is perturbed independently and by a different amount. Usually, the perturbed value of the primitive variable is obtained by a certain factor of the standard deviation on either side of the mean value. It is important to note that in the NESSUS code a linear buckling analysis is carried out by making use of the subspace iteration technique to evaluate the probabilistic buckling load. The matrix equation for the buckling (eigenvalue) analysis for a linear elastic structure is as follows:

$$\{[K] - \lambda\,[K_g]\}\{\phi\} = 0 \tag{1}$$

Finally, the NESSUS/fast probability integration (FPI) module extracts eigenvalues to calculate a probability distribution of the eigenvalues and to evaluate respective sensitivities associated with the corresponding uncertainties in the primitive variables. The mean, distribution type, and percentage variation for each of the primitive variables are given in Table 1. For example, the standard deviation for the vertical load was 1 lb and was perturbed on each side of its mean value by 1.26 × the standard deviation.

Probabilistic Progressive Buckling Computational Simulation

Initially, the truss was deterministically analyzed for member forces and to identify the members in which the axial forces exceed the Euler load. The Euler buckling load check is used only to identify the members that are the most likely (have the highest probability) to buckle so that a finer mesh can be used to obtain the buckled load and shape. This is a computationally efficient way to handle the complex problem. As mentioned earlier, at the truss fixed end (left side) panel points, all six degrees of freedom were restrained, whereas other panel points were not. Thus the member axial forces are dependent on the individual member end node boundary conditions. However, an individual member's local Euler buckling load was calculated based on the assumption that the member was pin ended and was then compared with the actual axial force to determine whether the axial force exceeded the local Euler buckling load. These members were then discretized with several intermediate nodes to obtain an actual defleceted shape of the members. The subsequent buckling (eigenvalue) analysis was performed to probabilistically evaluate buckling loads and respective buckled shapes. Furthermore, the sensitivity factors representing the impact of uncertainties in the primitive variables on the scatter of response variable (eigenvalue) are evaluated. Finally, any members that have buckled are identified, and the probabilistic buckled loads/moments at each probability level are obtained by multiplying the respective eigenvalues with the applied loads and moments.

In the subsequent analyses the buckled members are removed from the original truss configuration, and the previously described analysis steps are repeated until onset of the collapse state is reached. It is important to note that the mean values of the loads and moments are kept constant and are perturbed

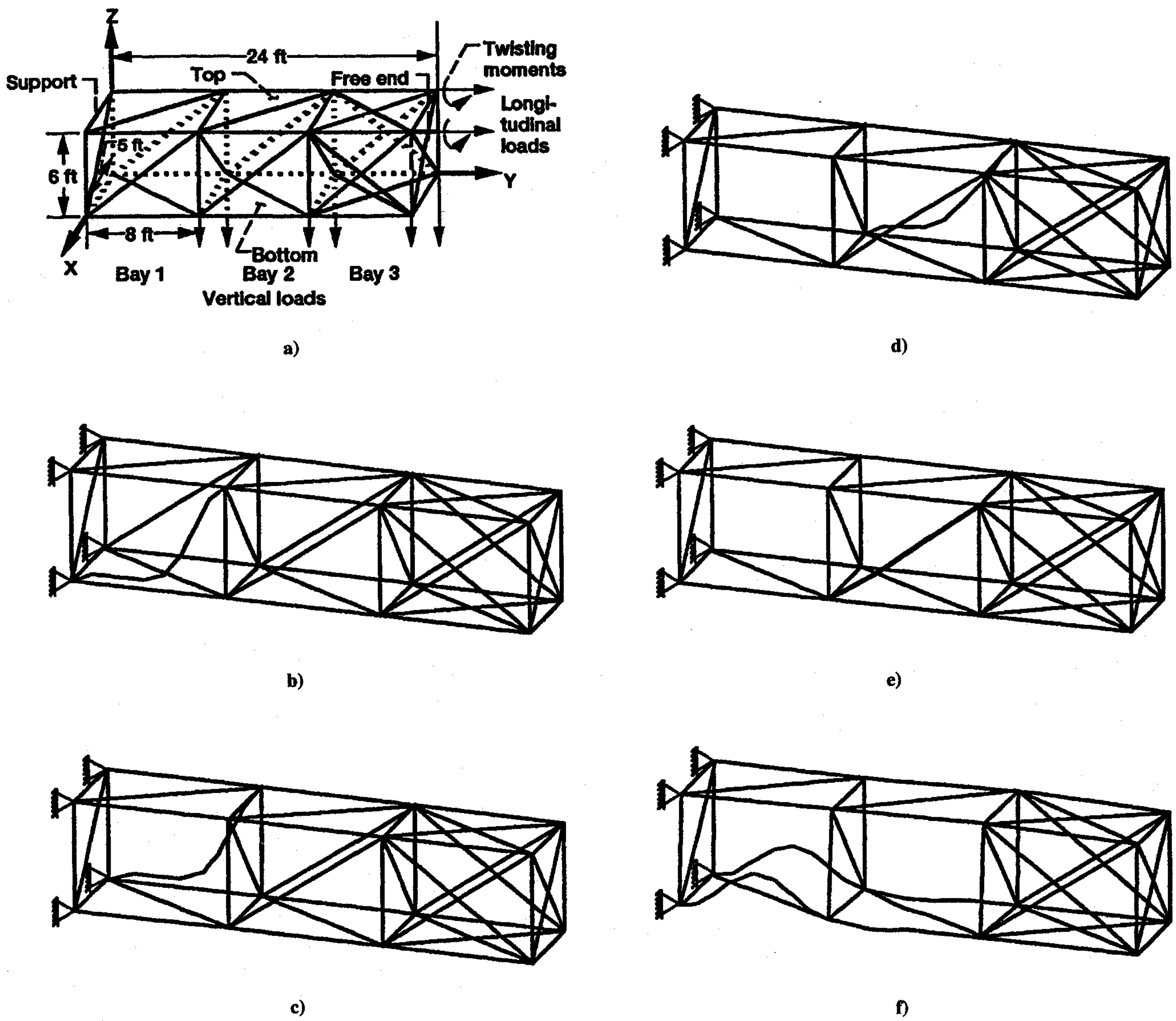

Fig. 2 Probabilistic progressive buckling as buckled members are sequentially removed.

around their means during the probabilistic buckling analysis. The truss end node displacements vs the number of members removed are plotted to identify the onset of the truss collapse state. Finally, the minimum number of members needed to support the applied loads and moments are determined. It is worth noting that the progressive buckling described herein is unique to this research, and the authors are not aware of any literature on that subject.

Discussion of Results

Probabilistic Progressive Buckling—First Buckled Member

Figures 2a–2f show the probabilistic progressive buckled mode shapes of the three-bay space truss as individual buckled members are sequentially removed from the original configuration until it reaches the onset of collapse. The probabilistic buckling analysis indicated that the first bay front diagonal buckled first (Fig. 2b), and the corresponding probabilistic buckled loads and moments at 0.5 probability are shown for example in Fig. 3. Probabilistic buckled loads and moments at different probability levels can also be obtained. Furthermore, a method of calculating the margin of safety (MOS) for specified probability by using known distributions for applied loads and moments and the corresponding cumulative distribution function curves obtained from PSAM are shown in Fig. 4. The MOS is used in deterministic designs to obtain an assessment on the additional margin of safety available that is beyond those used in the design as the safety factor for loads and for stress allowables. MOS also provides a measure of design robustness. However, it does not reflect the probability of failure or the reliability of the design. It is used herein in the same context but for a specified probability of the buckling event. The sensitivity factors from Fig. 5 suggest that the scatter in the bay length parameter (Y coordinate) had the highest impact on the probabilistic distribution of the buckling load followed by the bay height (Z coordinate), bay width (X coordinate), vertical and longitudinal loads, and finally twisting moments. Any slight variation in spacial (geometry) variables has a direct effect on the overall length of the members and thereby alters many terms in the stiffness matrix containing the length parameter. Finally, this has a definite effect on the probabilistic buckling loads, which has been clearly observed in the previously discussed results. However, it is important to note that even comparatively large variations in both member modulus E and area r_o and r_i (see Table 1) had very negligible impact. Similar conclusions can also be drawn for the probabilistic member force in the first buckled member (see Fig. 6). The variation in the resistance (mean area × mean yield strength) of the

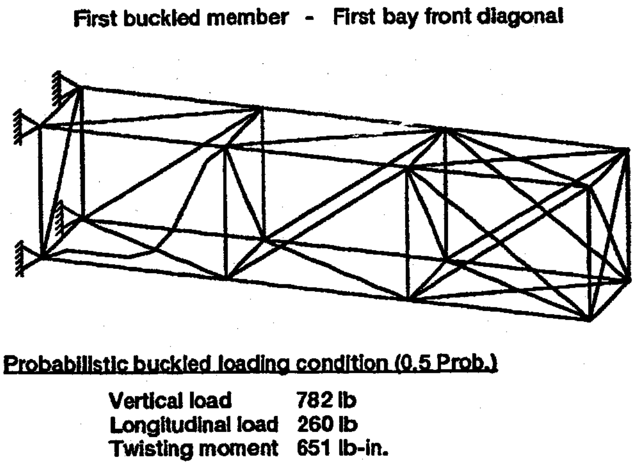

Fig. 3 Probabilistic progressive buckling.

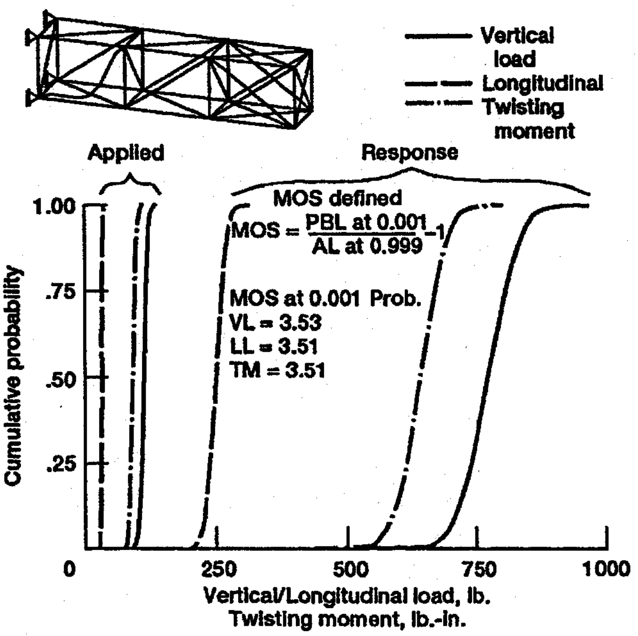

Fig. 4 Probabilistic buckling load.

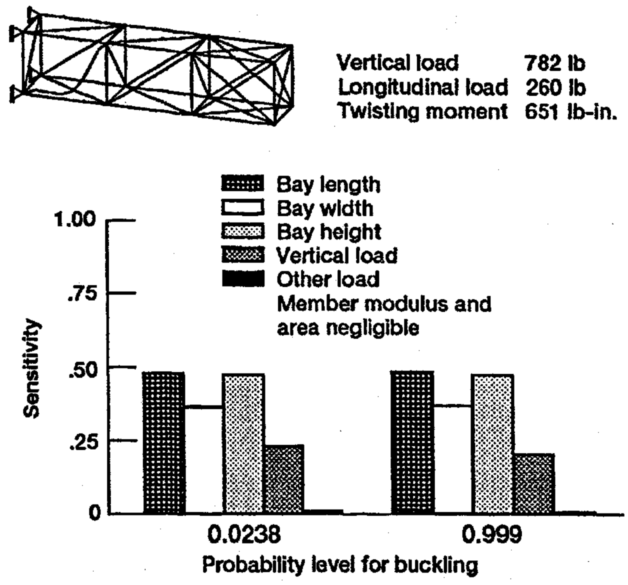

Fig. 5 Probabilistic buckling load sensitivity.

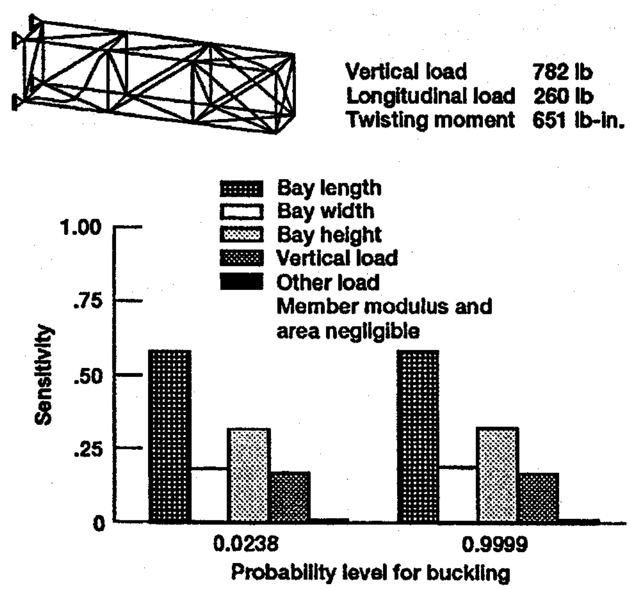

Fig. 6 Probabilistic member force sensitivity.

member was assumed to have a Weibull distribution and is shown in Fig. 7. MOS calculations for strength exceedence using distribution curves for probabilistic member force and resistance as well as probabilistic buckling load and resistance indicate that the buckled member did satisfy the strength criteria condition. Therefore, it can be concluded from Figs. 4 and 7 that the member buckled when its axial force exceeded the Euler buckling load and when the stress due to this load did not exceed the failure criteria.

Probabilistic Progressive Buckling—Second, Third, and Fourth Buckled Members

As described in the previous section, the deterministic analysis followed by the probabilistic analysis was performed with sequential removal of the first, second, third, and fourth buckled members from the truss, and the probabilistic buckled loads and moments, sensitivity factors, and MOS values for stress were obtained. When the second member was buckled (see Fig. 2c), the comparable results as shown in Figs. 4–7 are described in Figs. 8–11.

For these truss configurations the MOS value decreased from 3.53 to 2.53. The similar details of the truss with the third buckled member (see Fig. 2d) are given in Figs. 12–15. According to Fig. 13, the MOS value further decreased to 1.62. Similarly, Figs. 16–19 give comparable results of the truss when the fourth member was buckled (see Fig. 2e). It is important to note from Figs. 18 and 19 that the scatter in the bay height had much higher impact than scatter in both bay width and length on both the probabilistic buckling loads/moments and buckled member force. Finally, the details of the onset of the collapse state of the truss (Fig. 2f) are shown in Figs. 20–22. When all of the four buckled members were removed, the MOS value was equal to −3.75, which indicates that the onset of collapse was reached (Fig. 21). Furthermore, the probabilistic buckling loads/moments at 0.001 probability were equal to maximum applied loads/moments with assumed distributions (see Fig. 20). In addition, at the collapse state the uncertainties in both the bay length and bay height had sufficiently high impact on the probabilistic buckling loads/moment distributions (see Fig. 22). In the previously discussed various truss configurations, the uncertainties in the vertical loads had consistently the same impact on buckling loads/moment, whereas member modulus and area had a negligible impact.

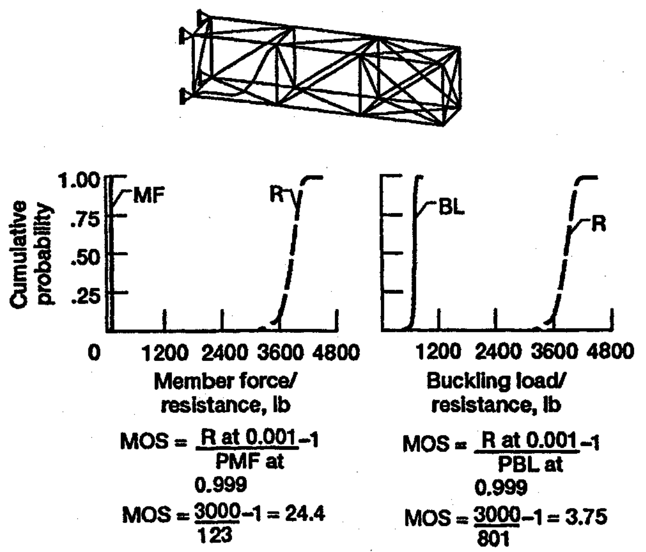

Fig. 7 Probability of strength exceedence (for first bay front diagonal).

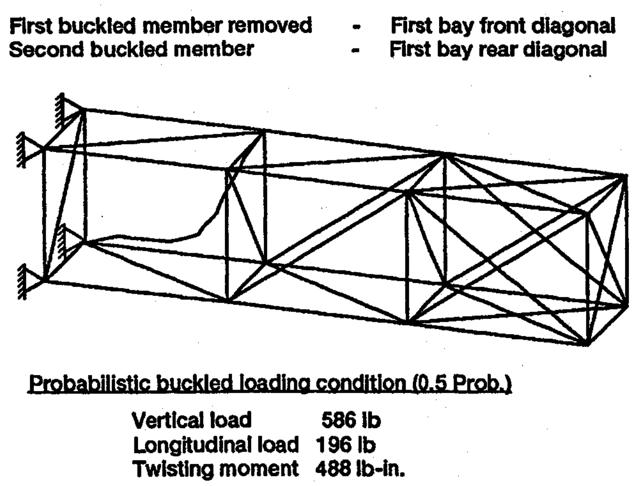

Fig. 8 Probabilistic progressive buckling.

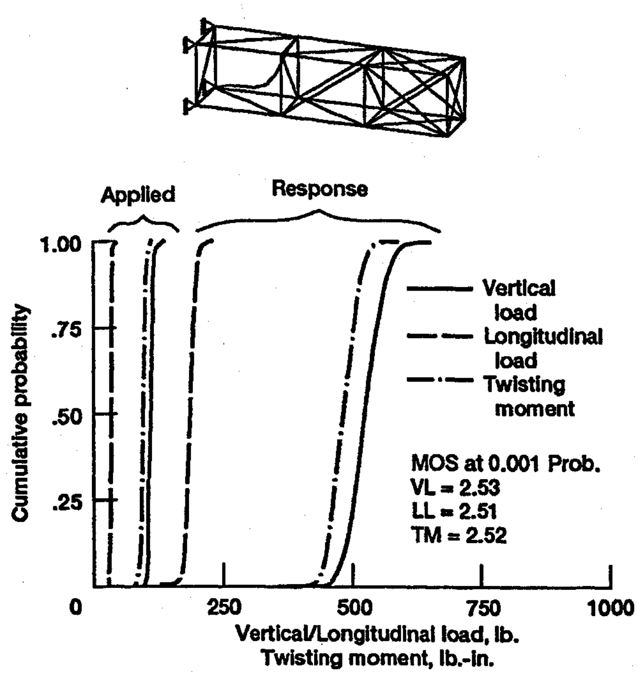

Fig. 9 Probabilistic buckling load.

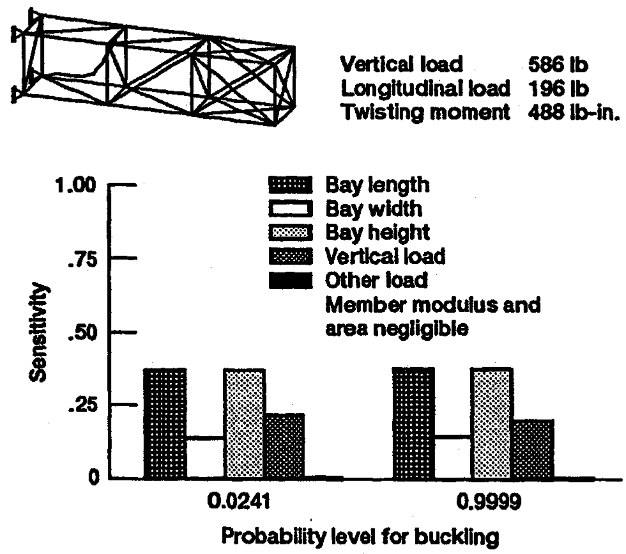

Fig. 10 Probabilistic buckling load sensitivity.

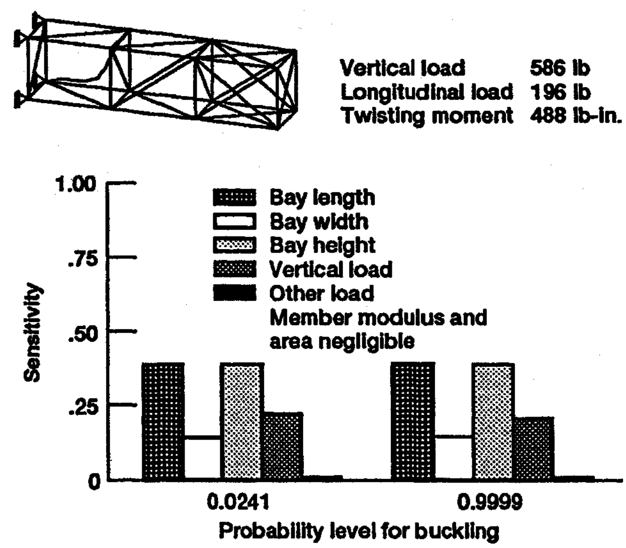

Fig. 11 Probabilistic member force sensitivity.

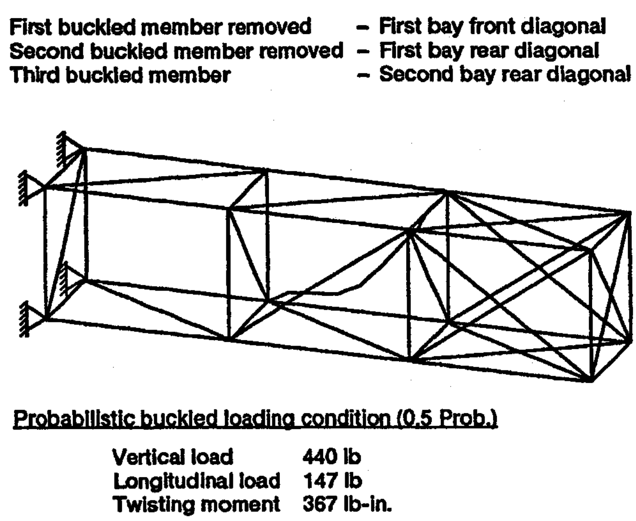

Fig. 12 Probabilistic progressive buckling.

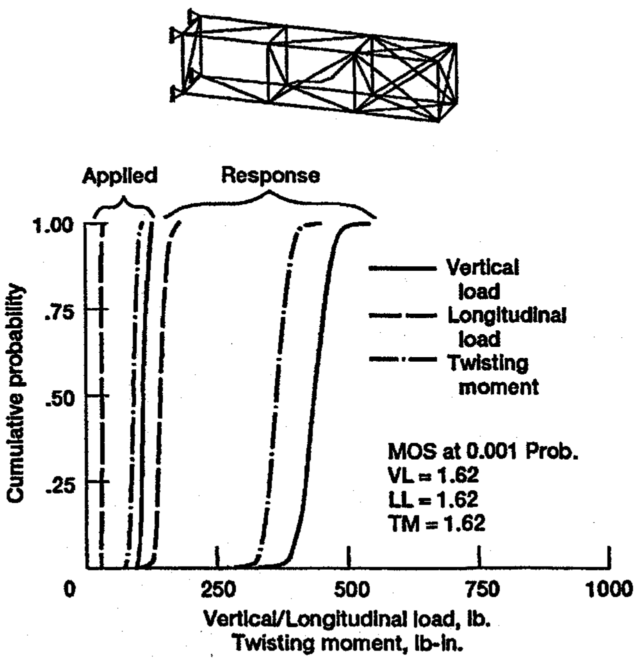

Fig. 13 Probabilistic buckling load.

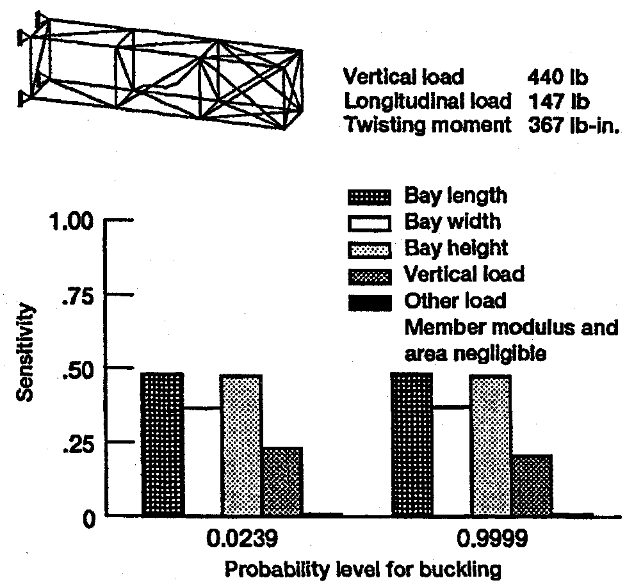

Fig. 14 Probabilistic buckling load sensitivity.

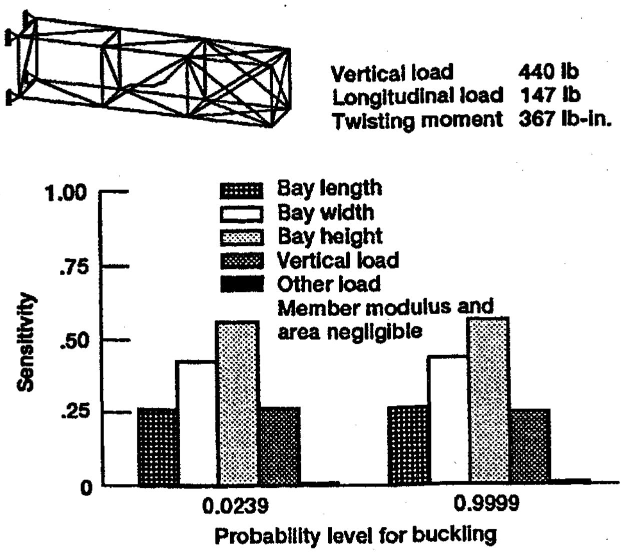

Fig. 15 Probabilistic member force sensitivity.

First buckled member removed - First bay front diagonal
Second buckled member removed - First bay rear diagonal
Third buckled member removed - Second bay rear diagonal
Fourth buckled member - Second bay front diagonal

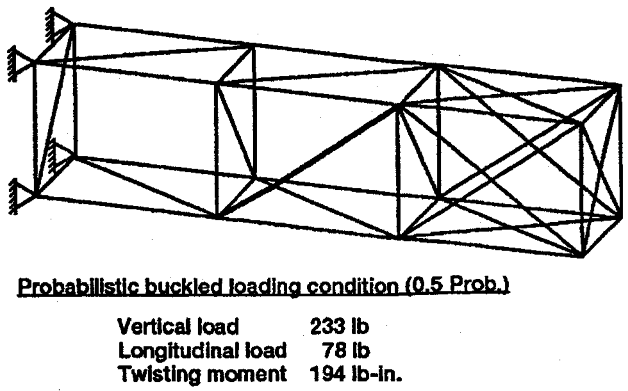

Fig. 16 Probabilistic progressive buckling.

Probabilistic Truss End Node Displacements

The truss end node displacements (lateral, longitudinal, and lateral) were also calculated during each of aforementioned deterministic analyses for each truss configuration and are shown in Fig. 23. It is clear that there is not considerable change in either lateral or longitudinal displacement as each buckled member was sequentially removed. However, the truss end node vertical displacement gradually increased up to the truss configuration with three buckled members removed and suddenly increased very rapidly when the fourth buckled member was removed, giving an indication of unbounded displacement growth that suggests that the truss had reached the onset of its collapse state. This is because the total vertical loads are three times the total longitudinal loads (Table 1) and the perturbations for the vertical loads were arbitrarily chosen much higher compared with the longitudinal loads in view of analyzing their effects as they seemed to be more significant. Furthermore, the twisting moments were much smaller compared with other loads. However, it is important to note that, for a realistic design, prudent judgment should be used for selecting the perturbation sizes. Figures 24 and 25, respectively, show the relationships between the applied vertical loads and probabilistic buckling loads as well as probabilistic buckling loads and MOS values. The optimum truss configuration was reached with the fourth buckled member removed whereby the probabilistic buckling load was equal to the applied vertical load at the 0.001 probability level (see Fig. 24). Similar conclusions can also be made for longitudinal loads and twisting moments. In addition, there is a gradual decrease in the MOS values as buckled members were sequentially removed and reached a zero value when the optimum truss configuration was reached (see Fig. 25). Similar conclusions can also be made for longitudinal loads and twisting moments.

Probabilistic Buckling Including Initial Eccentricity

In the previously discussed probabilistic progressive buckling methodology, all of the members were assumed to be initially perfectly straight, and the buckled members were sequentially removed with the assumption that once the member buckled it would yield and could not resist any additional loading and thereby would not contribute to the overall stiffness of the truss.

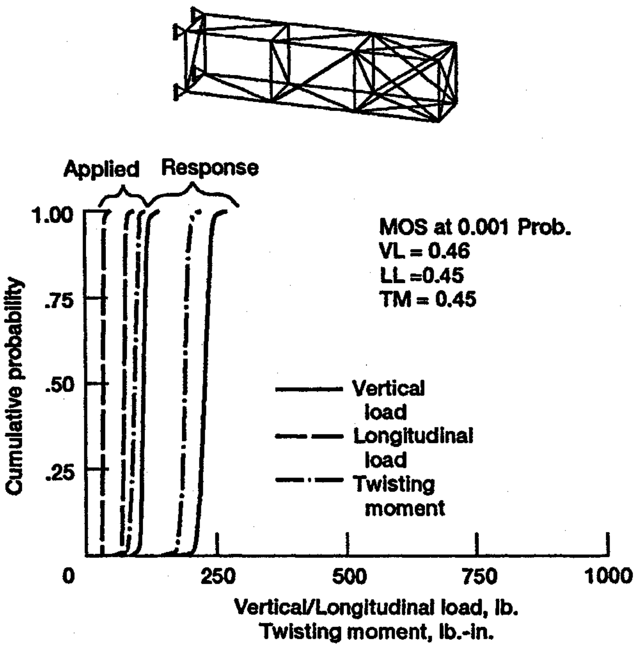

Fig. 17 Probabilistic buckling load.

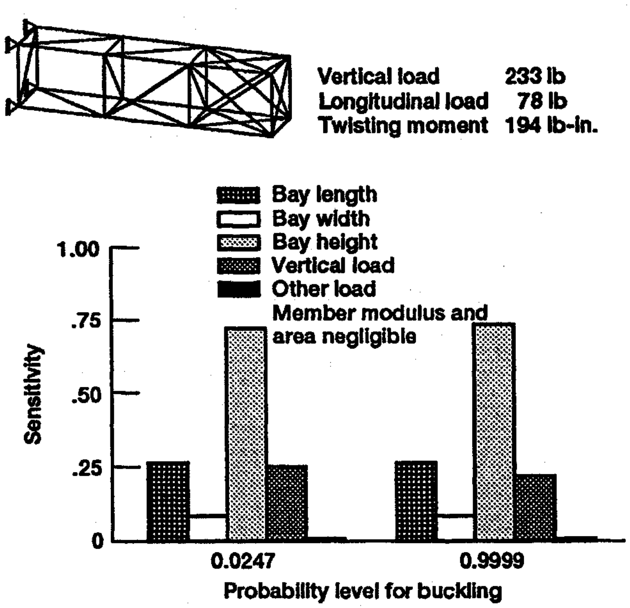

Fig. 18 Probabilistic buckling load sensitivity.

To verify this assumption, the maximum eccentricity at which the yielding in the member (first bay front diagonal) would take place due to the combined effects of axial and in-plane bending moments was calculated. Furthermore, this member was modeled to depict the buckled configuration of the member at which yielding will take place, using a parabolic distribution for the previous calculated eccentricity (see Fig. 26). The deterministic and subsequent probabilistic buckling analyses indicate, respectively, that the probabilistic buckling loads and moments did not change significantly from the original analysis (see Fig. 3) and the first bay rear diagonal has buckled (see Fig. 27). However, as seen from Figs. 5 and 28 for probabilistic buckling loads and from Figs. 6 and 29 for probabilistic member forces, the sensitivity factors show some changes; especially the variations in bay width have the most dominant impact on both probabilistic buckling loads and moments (see Figs. 28 and 29). This is because the member buckles in the plane perpendicular to the direction of the loading. Nevertheless, it is important to note that the scatter in the spacial location

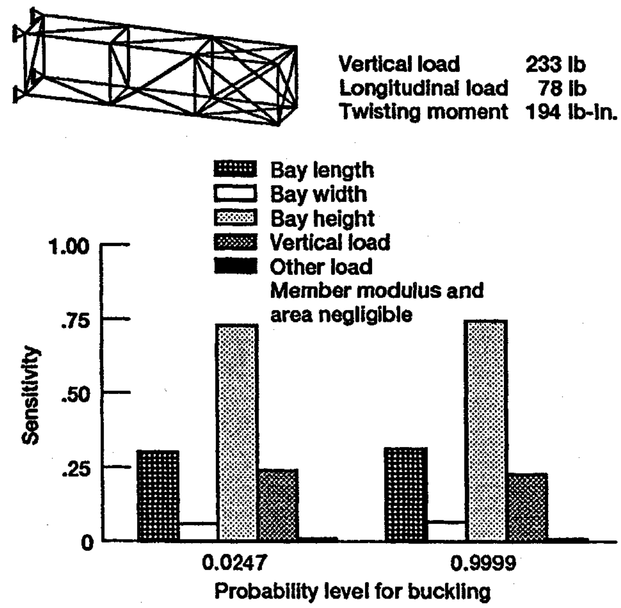

Fig. 19 Probabilistic member force sensitivity.

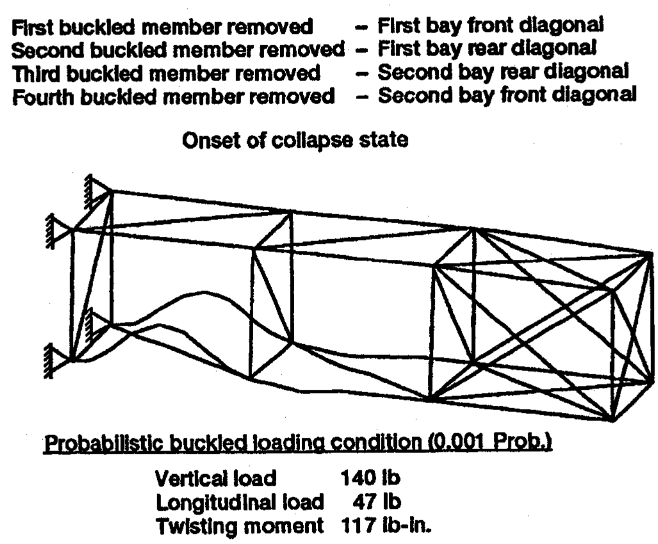

Fig. 20 Probabilistic progressive buckling.

accentuates the sensitivities of the bay length/width/height on the probabilistic load and diminishes that of vertical load. Once again the variations in the member modulus and area have very negligible impact. These results justify the sequential removal of the buckled members during progressive buckling.

Conclusions

The computational simulation of probabilistic progressive buckling of a space-type truss is demonstrated using the NESSUS computer code, and the step-by-step procedure is outlined. The methodology for onset of truss collapse is established. The progressive buckled mode shapes are obtained, and sensitivities associated with the uncertainties in the primitive variables are evaluated. The safety margins are determined, and the optimum truss is obtained for the given loading condition. The results indicate that 1) probabilistic buckling loads and margin of safety values decrease as buckled members are sequentially removed, 2) the scatter in truss geometry (bay length/width/height) and vertical loads has considerable impact on the probability of the buckled load, 3) the member modulus and area parameters have negligible impact, and 4) initial eccentrics have a negligible influence on the probabilistic buckling load but may influence the sensitivities. Collectively the results demonstrate that the

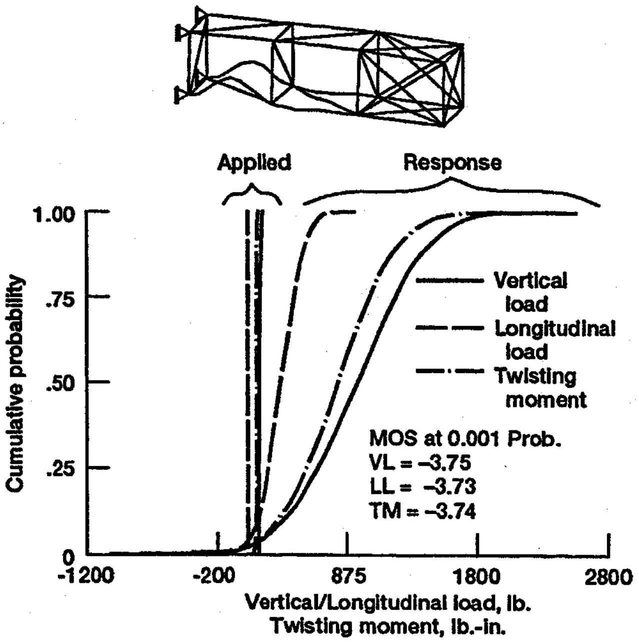

Fig. 21 Probabilistic buckling load.

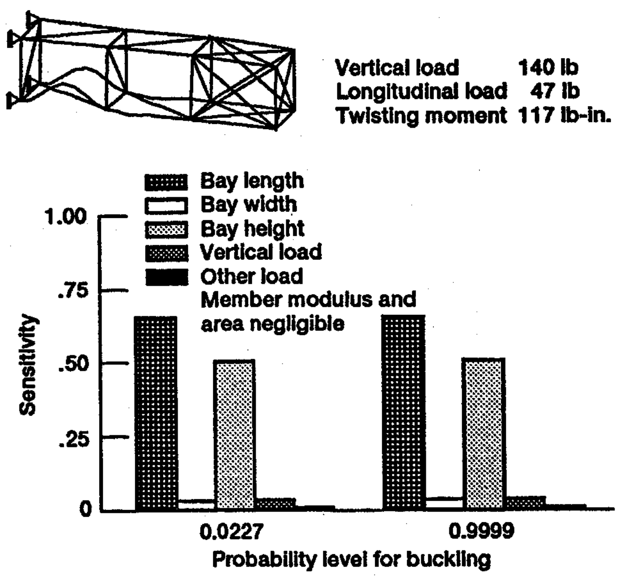

Fig. 22 Probabilistic buckling load sensitivity.

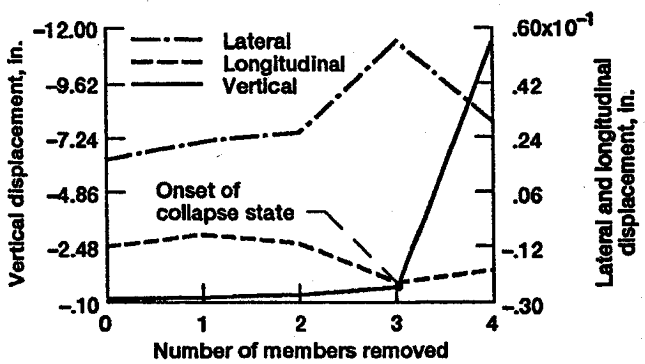

Fig. 23 Progressive buckling leading to structural collapse as indicated by unbounded displacement.

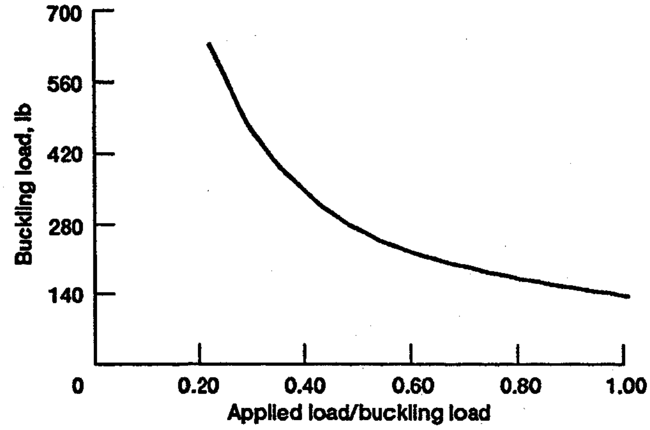

Fig. 24 Buckling load vs applied load / buckling load at 0.001 probability.

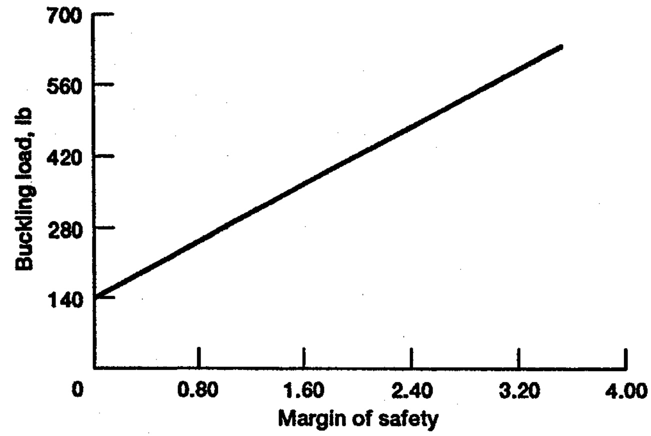

Fig. 25 Buckling load vs margin of safety at 0.001 probability.

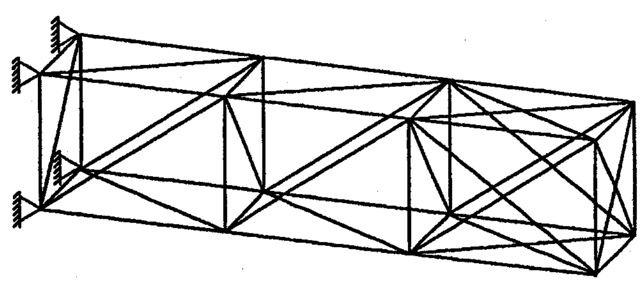

Fig. 26 First bay front diagonal with initial eccentricity.

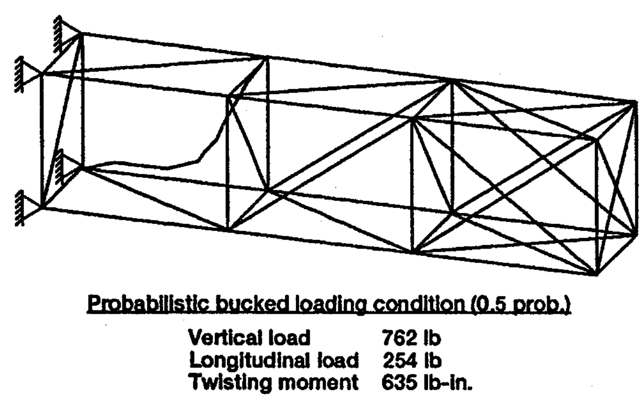

Fig. 27 Probabilistic buckling—first bay rear diagonal buckled.

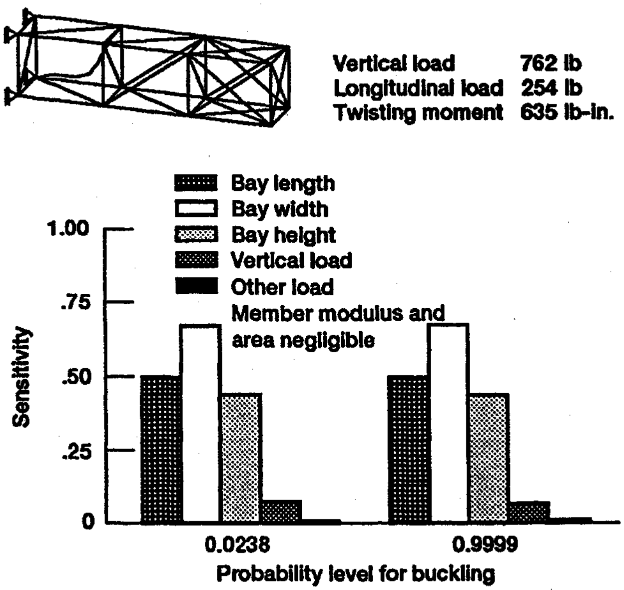

Fig. 28 Probabilistic buckling load sensitivity.

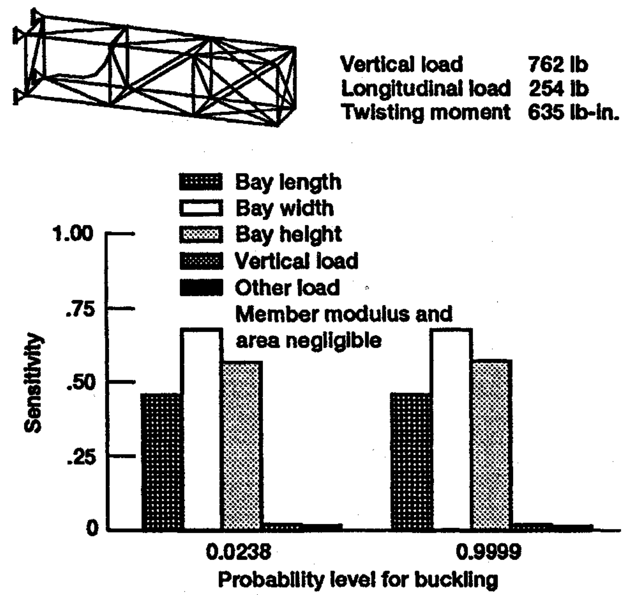

Fig. 29 Probabilistic member force sensitivity.

probability of collapse of space-type trusses can be reliably assessed by the procedure described herein.

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